

## Conformational Properties of Liquid Polyethylene

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**ABSTRACT:** The conformational entropy  $S_c$  of linear polyethylene liquid is calculated at  $T_m = 414$  K for the three-state rotational model taking into account the "pentane interference". The volume contribution  $S_v$  is estimated using a volume-dependent  $(\partial p/\partial T)_v$ , which is derived via the Tait equation for liquid compressibility. The values are (cal deg<sup>-1</sup> mol<sup>-1</sup> CH<sub>2</sub>)  $S_c = 1.295$  and  $S_v = 1.057$ , and the sum is in excellent agreement with the experimental fusion entropy for extended-chain crystals:  $\Delta S_m = 2.362$ . Previous studies using isolated or unperturbed chain values for the rotational energy have over-estimated  $S_c$  and based on a volume-independent  $(\partial p/\partial T)_v$  have under-estimated  $S_v$ . From a consideration of the pressure dependence of  $T_m$  up to 2000 bars, it is found that both  $S_c$  and  $f$ , the fraction of flexed bonds, remain constant at  $T_m$  while  $\Delta S_m$  and  $S_v$  decrease. The "conformational volume"  $V_c$  associated with the rotational isomerization is estimated to be  $V_c = 4.31$  cm<sup>3</sup>/mol of flexed bonds. At  $T_m$ ,  $V_c$  constitutes 30% of  $\Delta V_m$ . For statistical mechanical calculations of the conformational properties, the simple rotational partition function due to Temperley is shown to be an adequate approximation to a more complex function derived later.

In an earlier paper,<sup>1</sup> relationships were developed between the conformational entropy  $S_c$  of polymer liquids and  $T_0$ , the constant in the Vogel equation for polymer liquid mobility. These were based on a modification of the Adam–Gibbs theory<sup>2</sup> for cooperative relaxation. In this new interpretation, the conformational entropy,  $S_c$ , arising from rotational isomerization about main-chain bonds, rather than the excess entropy  $\Delta S = S(\text{liquid}) - S(\text{crystal})$ , is the critical parameter in determining liquid mobility. In that work,<sup>1</sup> the simplest three-state rotational model with independent bond rotations was adopted.

In the present paper and with linear polyethylene (PE) as an example, we consider a physically more realistic model which takes into account the "pentane interference" and derive the conformational contribution to  $\Delta S_m$ , the entropy of fusion. The volume contribution  $S_v$  is calculated on the basis of a volume-dependent thermal-pressure coefficient  $(\partial p/\partial T)_v$ , which in turn is derived via the Tait equation for liquid compressibility. The effects of pressure on  $\Delta S_m$  and  $S_c$  are related to the pressure dependence of  $T_0$  (Vogel) and from the latter the "conformational volume"  $V_c$ , associated with the rotational isomerization, is estimated.

## General Equations

We adopt the three-state rotational model (a trans state and two gauche states at a higher energy  $U$ ) in which transitions between gauche bonds of opposite sign ( $g^+g^-$ ) are excluded (the "pentane interference"). For this model the partition function is:<sup>3,4</sup>

$$Q = 0.5[1 + \sigma + (1 + 6\sigma + \sigma^2)^{1/2}] \quad (1A)$$

in which  $\sigma = \exp(-U/RT)$ .

This superseded the simpler function derived earlier by Temperley:<sup>5</sup>

$$Q' = [1 + 2^{1/2} \exp(-U/RT)] \quad (1B)$$

For a reason which will be given shortly, both of these functions will be developed in parallel. By standard statistical thermodynamics, the conformational (i.e., rotational) entropy is given by  $S_c = R \ln Q + RT (\partial \ln Q/\partial T)$ , which for the two partition functions being considered leads to:

$$S_c/R = \ln Q + \frac{(U/RT)(Q + 1)\sigma}{Q(2Q - 1 - \sigma)} \quad (2A)$$

or

$$S_c'/R = \ln Q' + (U/RT)(Q' - 1)/Q' \quad (2B)$$

These are both plotted in Figure 1. By the indicated linear extrapolations and the arguments given previously,<sup>1</sup>

$$U = 4.4RT_0 \quad (3A)$$

or

$$U' = 4.0RT_0 \quad (3B)$$

As before,<sup>1</sup> the conformational entropy can also be expressed by the simple empirical equation:

$$S_c = 2.1(T - T_0)/T \quad (4A)$$

which applies between the limits  $T_0/T \simeq 0.28$  to 0.68, the linear portion of curve A in Figure 1. For the alternate case,

$$S_c' = 2.0(T - T_0)/T \quad (4B)$$

which applies between the limits  $T_0/T \simeq 0.19$  to 0.69 (curve B).

The fraction of flexed bonds in the  $g^+, g^-$  states is given by:

$$f = (Q + 1)\sigma/Q(2Q - 1 - \sigma) \quad (5A)$$

or

$$f' = (Q' - 1)/Q' \quad (5B)$$

Compared to eq 2A, the differentiation of eq 2B with respect to temperature is considerably more tractable. Fortunately, as will be shown later, both equations give very similar values for  $S_c$  and its temperature derivative, and hence, as a very good approximation, the conformational heat capacity can be derived from eq 2B:

$$C_{pc} = T(\partial S_c/\partial T)_p = R(U/RT)^2(Q' - 1)/Q^2 \quad (6B)$$

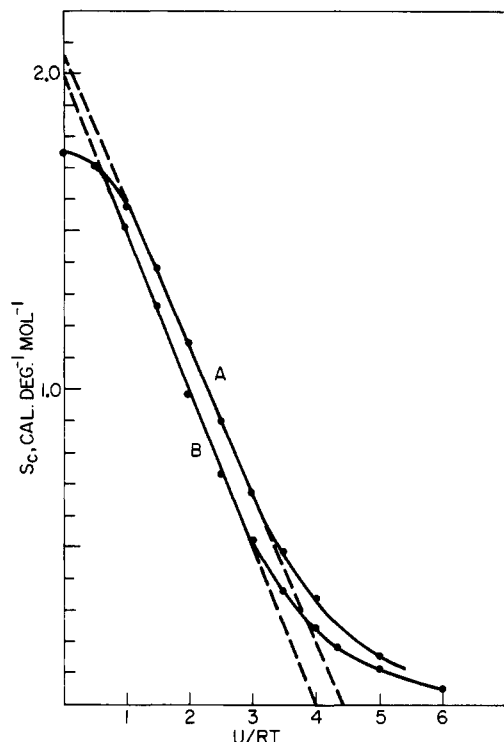
Also<sup>1</sup>

$$-(\partial S_c/\partial p)_T = C_{pc} (\partial \ln U/\partial p)_T \quad (7B)$$

The volume entropy  $S_v$  is obtained from the thermodynamic identity,

$$(\partial S/\partial v)_T = (\partial p/\partial T)_v = \gamma \quad (8)$$

Contrary to the assumption that has often been adopted in estimates of  $S_v$  at the melting points of crystalline polymers,<sup>6-8</sup> it is now well established that  $\gamma$  is not constant but increases as the liquid is compressed isothermally.<sup>9-11</sup> Owing to the pressure-induced crystallization, the isothermal change in  $\gamma$  with liquid volume at  $T_m$  cannot be measured directly but has been estimated by a linear



**Figure 1.** Conformational entropy for the three-state rotational model ( $g^\pm g^\mp$  transitions excluded) with differing rotational partition functions: (A) eq 2A; (B) eq 2B.

extrapolation of  $\gamma$  measured in the melt above  $T_m$ .<sup>10,11</sup>

We have proposed a new method for deriving  $\gamma$  as a function of volume in the isothermal compression of a polymer liquid.<sup>12</sup> This is based on the Tait relationships for liquid compressibility:

$$V = V_0[1 - c \ln(P + b)/b] \quad (9)$$

$$\beta = -(\partial \ln v / \partial p)_T = V_0 C / V(P + b) \quad (10)$$

in which  $V_0$  and  $V$  are the liquid specific volumes at  $P_0$  (atmospheric pressure) and pressure  $P$ , respectively, at the same temperature, and  $C = 0.0894$ , a universal constant which is independent of pressure and temperature.<sup>13-15</sup> The temperature dependence of  $b$  is:<sup>15-17</sup>

$$b = b_0 \exp(-b_1 T) \quad (11)$$

or  $-d \ln b / dT = b_1$ .

The thermal expansion coefficient,  $\alpha$ , as a function of pressure is given by the temperature derivative of eq 9 at constant pressure:<sup>15,17</sup>

$$(\partial \ln v / \partial T)_p = \alpha = \alpha_0 - P\beta b_1 \quad (12)$$

where  $\alpha_0$  is the value at  $P_0$ . Thus  $\alpha$  and  $\beta$  can be calculated for a series of pressures (or volumes) from which  $\gamma = (\alpha/\beta)$  is derived as a function of volume at constant temperature.

## Results

**$S_v$  and  $S_c$  at  $T_m$ .** Wunderlich and Czornyj have recently presented a critical review of the equilibrium melting of linear polyethylene.<sup>18</sup> For extended-chain crystals at  $T_m = 414$  K,  $V_s = 1.034$  cm<sup>3</sup>/g for the crystal specific volume. For the liquid at  $T_m$ , we take the value measured by Olabisi and Simha,<sup>16</sup>  $V_0 = 1.268$  cm<sup>3</sup>/g. From the latter work and eq 11,  $b(T_{\text{ait}}) = 916$  bars. The calculations for  $\gamma$  as a function of volume are listed in Table I, and the results are plotted in Figure 2. A "graphical integration" of this nonlinear plot was performed by summation of the product of the average  $\bar{\gamma}$  for each

**Table I**  
Calculation of  $\gamma$  at  $T_m = 414$  K

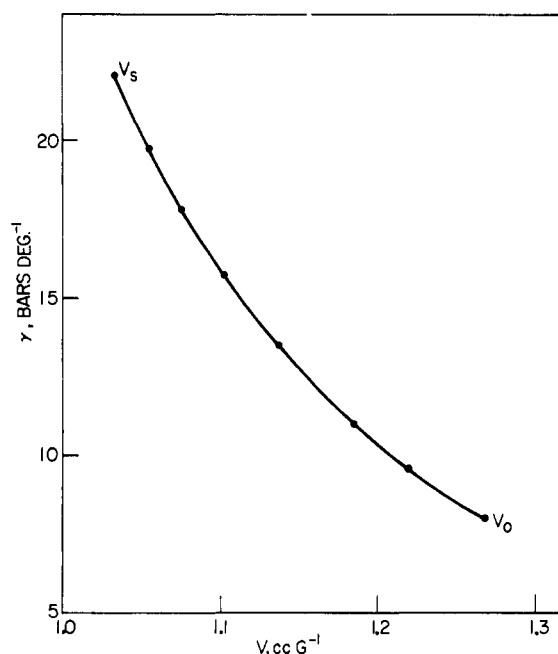
$P$ , kbar	$V$ , cm <sup>3</sup> /g	$10^5 \beta$ , bar <sup>-1</sup>	$10^4 \alpha$ , deg <sup>-1</sup>	$\gamma$ , bar/deg
0	1.268 <sup>a</sup>	9.75 <sup>b</sup>	7.81 <sup>c</sup>	8.01 <sup>d</sup>
0.5	1.219	6.57	6.27	9.55
1.0	1.184	5.00	5.48	10.96
2.0	1.137	3.42	4.62	13.50
3.0	1.103	2.62 <sub>5</sub>	4.14	15.76
4.0	1.077 <sub>5</sub>	2.14	3.81 <sub>5</sub>	17.82
5.0	1.056 <sub>5</sub>	1.81	3.58	19.72
6.3	1.034	1.52	3.34	22.00

<sup>a</sup> Equation 9,  $b = 916$  bars. <sup>b</sup> Equation 10. <sup>c</sup> Equation 12. <sup>d</sup>  $\gamma = \alpha/\beta$ .

**Table II**  
Entropy Contributions<sup>a</sup> at  $T_m = 414$  K ( $T_0 = 160$  K)

$U$ , cal/mol	1335 <sup>b</sup>	1271 <sup>c</sup>	1398 <sup>d</sup>
$S_c$	1.573 <sup>b</sup>	1.234 <sup>c</sup>	1.295 <sup>d</sup>
$(S_c + S_v)^e$	2.630	2.291	2.352

<sup>a</sup>  $\Delta S_m = 2.362 (\pm 0.115)$ ; <sup>18</sup> all entropy units are cal deg<sup>-1</sup> mol<sup>-1</sup> CH<sub>2</sub>. <sup>b</sup>  $g^\pm g^\mp$  allowed, eq 7 and A3 in ref 1. <sup>c</sup>  $g^\pm g^\mp$  excluded, eq 3B and 2B. <sup>d</sup>  $g^\pm g^\mp$  excluded, eq 3A and 2A. <sup>e</sup>  $S_v = 1.057$ , this work.



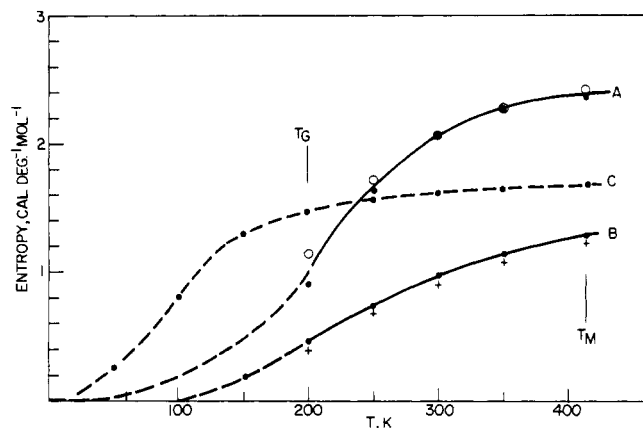
**Figure 2.** Volume dependence of the thermal pressure coefficient  $\gamma$  at  $T_m = 414$  K.

pressure interval in Table I and the volume increment  $dv$  over the same interval. Thus,

$$S_v = \sum_{V_s}^{V_0} \bar{\gamma} dv = 3.160 \text{ cm}^3 \text{ bars deg}^{-1} \text{ g}^{-1} =$$

$$1.057 \text{ cal deg}^{-1} \text{ mol}^{-1} \text{ CH}_2$$

By an indirect method involving extrapolations from two lower  $n$ -alkanes, Robertson<sup>9</sup> derived an expression for  $\gamma_\infty$  (referring to the infinite chain) which was linear in  $V$ . For our values of  $T_m$ ,  $V_0$ , and  $V_s$ , Robertson's eq 10 leads to  $S_v = 1.10$  cal mol<sup>-1</sup> deg<sup>-1</sup>, a remarkable agreement considering the vastly different procedures. By a linear extrapolation of  $\gamma - V$  from the melt in the vicinity of  $T_m$ , Tsujita and co-workers<sup>10</sup> reported  $S_v = 0.91$  cal mol<sup>-1</sup> deg<sup>-1</sup>, but for the lower values,  $T_m = 407$  K and  $\Delta V_m = 0.212$  cm<sup>3</sup>/g, presumably referring to ordinary folded-chain rather than extended-chain crystals. A linear extrapolation



**Figure 3.** Temperature dependence of entropies. (A) Excess entropy,  $\Delta S$ : (●) eq 13 and 14; (○) calculated by Broadhurst.<sup>22</sup> (B) Conformational entropy,  $S_c$ : (●) from eq 2A ( $U = 1398$ ); (+) from eq 2B ( $U = 1271$ ). (C)  $S_c$  from eq 2A with  $U = 500$  cal/mol.

from  $V_0$  in Figure 2 would give  $\gamma \approx 15$  bar/deg at  $V_s$ , from which  $S_v \approx 0.90$  cal deg<sup>-1</sup> mol<sup>-1</sup>.

Table II lists the calculation of the conformational entropy  $S_c$  for PE at  $T_m = 414$  K, involving the two rotational models and the three partition functions with  $T_0$  (Vogel) = 160 K.<sup>19,20</sup>

In Table II, the sum ( $S_v + S_c$ ) exceeds the experimental entropy of fusion  $\Delta S_m$  for the model which allows  $g^+g^-$  transitions while the exclusion of these transitions gives good agreement with  $\Delta S_m$ , particularly as calculated with eq 2A. This is gratifying because the neglecting of the "pentane interference" is physically unrealistic in the first place.<sup>3-5</sup>

Lower values of  $U$ , applying to the "isolated" chain, would also give ( $S_v + S_c$ ) exceeding  $\Delta S_m$  with our value of  $S_v$ . For example, Smith<sup>3</sup> used  $U = 800$  and 540 cal/mol in eq 2A and obtained  $S_c = 1.59$  and 1.68, respectively, at  $T_m = 414$  K. Also, Tonelli<sup>8</sup> has reported  $S_c = 1.76$  and  $S_v = 0.46$ – $0.52$  cal mol<sup>-1</sup> deg<sup>-1</sup>, the latter values based on the assumption of a volume-independent  $\gamma$ , essentially the value at  $V_0$  in Table I. It appears that past work<sup>6-8</sup> has over-estimated  $S_c$  while under-estimating  $S_v$  by about the same amount. Further evidence for this is the behavior of  $S_c$  over a broad temperature range as discussed next.

**Effects of Temperature.** The excess entropy is given by:

$$\Delta S = \int_{T_2}^T \Delta C_p d \ln T \quad (13)$$

in which  $T_2 = T_0 = 160$  K for PE.<sup>19,20</sup> Heat capacities for the liquid and crystalline phases reported by Wunderlich<sup>21</sup> lead to:

$$\Delta C_p = 7.266 - 0.0178T \text{ cal deg}^{-1} \text{ mol}^{-1} \quad (14)$$

Figure 3 shows  $\Delta S$  as a function of temperature between  $T_g = 200$  K<sup>19,20</sup> and  $T_m = 414$  K, calculated by eq 13 and 14. Shown also are the  $\Delta S$  values calculated by Broadhurst<sup>19,22</sup> by extrapolation from lower  $n$ -alkanes. The asymptotic approach of  $\Delta S$  toward zero has been demonstrated by Smith.<sup>23</sup>

The  $S_c$  values derived from eq 2B closely parallel those given by the more correct eq 2A with essentially the same slopes,  $(\partial S_c / \partial T)_P = C_{pc} / T$ . This justifies the use of the simpler eq 2B, leading to eq 6B and 7B. Curve C was calculated from eq 2A for a lower value,  $U = 500$  cal/mol, generally accepted for the "isolated" or "unperturbed" chain.<sup>4,8,24</sup> It is seen that the lower  $U$  does not permit  $S_c$  to decrease rapidly enough with temperature, with the result that in the vicinity of 200 K and below  $S_c$  exceeds

**Table III**  
Effects of Pressure on Entropies<sup>a</sup> at  $T_m$

$P$ , bars	0	1000	2000
$T_m$ , K <sup>28</sup>	415	443	463
$T_2 (=T_0)$ , K	160	169 <sup>27</sup>	174 <sup>27</sup>
$\Delta S_m$ <sup>b</sup>	2.39 (2.4 <sup>c</sup> )	2.12 (2.1 <sup>c</sup> )	1.96
$U$ , cal/mol <sup>d</sup>	1398	1479	1523
$U/RT_m$	1.694	1.682	1.665
$S_c$ <sup>e</sup>	1.298	1.304	1.317
$f$ <sup>f</sup>	0.221	0.222	0.223

<sup>a</sup> All entropy units are cal deg<sup>-1</sup> mol<sup>-1</sup> CH<sub>2</sub>. <sup>b</sup> Equations 13 and 14. <sup>c</sup> Experimental values from ref 30.

<sup>d</sup>  $U = 4.4RT_0$  (eq 3A). <sup>e</sup> Equation 2A. <sup>f</sup> Fraction of flexed bonds by eq 5A.

$\Delta S$ , which is clearly impossible. Furthermore, this curve is virtually identical with one calculated by Tonelli<sup>25</sup> with  $U_s = 500$  and  $U_w = 2000$  cal/mol, the latter energy referring to  $g^+g^-$  transitions as against the complete exclusion of these transitions ( $U_w = \infty$ ) adopted in the present work. This is because the statistical weight  $\sigma = \exp(-U_s/RT)$  overwhelms  $\omega = \exp(-U_w/RT)$ , especially at the lower temperatures. For example, at 414 K,  $\sigma = 0.54$  and  $\omega = 0.088$ , and at 200 K,  $\sigma = 0.28$  and  $\omega = 0.0065$ . A value of  $U = 800$  cal/mol<sup>3</sup> also leads to a discrepancy in  $S_c$  vis-à-vis  $\Delta S$  in the low-temperature region. We consider the results in Figure 3 to be conclusive evidence that  $U$  values appropriate for the isolated or unperturbed chain cannot be used in statistical mechanical estimates of  $S_c$  or other conformational properties for the real polymer liquid.

**Effects of Pressure.** As pointed out by Naoki and Nose,<sup>26</sup> according to the thermodynamic relationship  $[\partial(C_p/T)/\partial p]_T = -(\partial^2 v / \partial T^2)_p$  if the  $V$ - $T$  isobars are linear,  $C_p$  will be independent of pressure. Examination of the isobars<sup>16</sup> for liquid PE, as well as for the solid below  $\sim 80$  °C, where the crystallinity remains constant, showed no detectable curvature so we tentatively assume that  $\Delta C_p$  (eq 14) is independent of pressure. Since  $T_2 = T_0$  for PE, both will have the same pressure dependence. We have recently calculated  $T_0$  as a function of pressure up to 2000 bars<sup>27</sup> and together with eq 13 and 14 can now calculate  $\Delta S_m$  as a function of  $T_m$ ,  $P_m$  for the extended-chain crystals.<sup>28</sup> As shown in Table III,  $\Delta S_m$  decreases with increasing  $T_m$ ,  $P_m$  in qualitative agreement with experiment.<sup>29,30</sup>

In Table III, the conformational entropy  $S_c$  and  $f$ , the fraction of flexed bonds in the  $g^+$  and  $g^-$  states, were derived from eq 2A and 5A, respectively, recalling that  $U = 4.4RT_0$  (eq 3A). Both  $S_c$  and  $f$  appear to remain constant at  $T_m$ ,  $P_m$  (the alternate equations 2B, 5B, and 3B also give constant  $S_c$  and  $f$  but at very slightly different values). Hence the volume contribution  $S_v = \Delta S_m - S_c$ , as well as  $\Delta S_m$  itself, decreases with increasing  $T_m$ ,  $P_m$ .

**Conformational Volume.** The flexing of bonds in the melting transition from the ordered crystalline state to the more disordered liquid must involve a "conformational volume"  $V_c$  contribution to  $\Delta V_m$ , analogous to the  $S_c$  contribution to  $\Delta S_m$ . This can be estimated from the thermodynamic relation:  $(\partial V_c / \partial T)_P = -(\partial S_c / \partial P)_T$  and eq 7B.

Since  $U$  by eq 3A or 3B is directly proportional to  $T_0$ ,  $(\partial \ln U / \partial P)_T = \partial \ln T_0 / \partial P$ . For PE liquid at atmospheric pressure, we have estimated<sup>27</sup>  $10^5 \partial \ln T_0 / \partial P = 8.14$  bar<sup>-1</sup>, which is independent of temperature at constant pressure.  $C_{pc}$  was calculated from eq 6B from 0 to 500 K and introduced into eq 7B, with the results shown in Figure 4.

At  $T_m = 414$  K,  $10^4 (\partial V_c / \partial T) = 2.06$  cm<sup>3</sup> g<sup>-1</sup> deg<sup>-1</sup>. Taking  $10^4 dV_l/dT = 9.9$  and  $10^4 dV_s/dT \approx 3.0$ ,<sup>16</sup> the temperature dependence of the excess volume is  $10^4 d\Delta V/dT \approx 6.9$ , of which 30% is contributed by  $\partial V_c / \partial T$ .

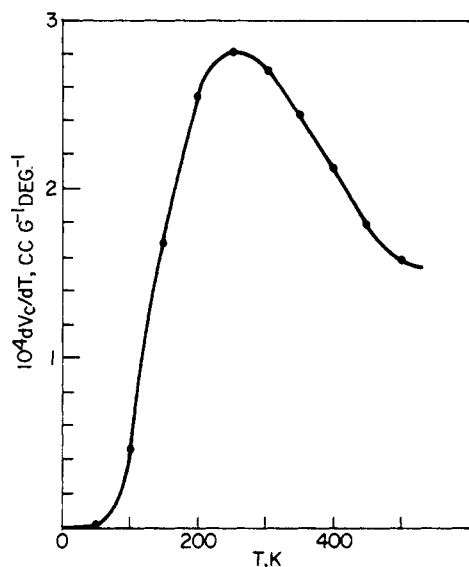


Figure 4. Temperature derivative of conformational volume as a function of temperature (eq 7B).

By graphical integration of Figure 4,  $V_c$  itself can be derived as a function of temperature. This is shown in Figure 5A. At  $T_m = 414$  K,  $V_c = 0.0715$   $\text{cm}^3 \text{g}^{-1}$ , which is again 30% of  $\Delta V_m = 0.234$   $\text{cm}^3 \text{g}^{-1}$ . In Figure 5B,  $V_c$  is plotted against  $f$ , the fraction of flexed bonds calculated by eq 5B. The relationship is now linear with a slope of  $0.308$   $\text{cm}^3 \text{g}^{-1}$  or  $4.31$   $\text{cm}^3$  per mol of flexed bonds. As a cross-check,  $f = 0.232$  at  $T_m$  and  $V_c = (4.31/14)f = 0.0713$   $\text{cm}^3 \text{g}^{-1}$ , in agreement with  $V_c$  derived above. The value  $V_c = 4.31$   $\text{cm}^3 \text{mol}^{-1}$  is to be compared with the van der Waals volume<sup>31</sup> of  $10.13$   $\text{cm}^3 \text{mol}^{-1}$  and the "hard-core" volume<sup>31</sup> at 0 K of  $13.40$   $\text{cm}^3$  per mol of  $\text{CH}_2$ . At the molecular level the flexing of a bond produces a volume increment of  $4.31/(6.02 \times 10^{23}) = 7.16 \times 10^{-24}$   $\text{cm}^3$ .

## Discussion

Karasz and co-workers<sup>32</sup> recently have questioned the entire concept of the separation of  $\Delta S_m$  into  $S_v$  and  $S_c$  components. In rebuttal, we point out that neither  $S_c$  nor  $S_v$  alone can account for  $\Delta S_m$ , but the two combined give a satisfactory entropy balance, as shown in the present work. Furthermore, their arguments do not apply to the general case of noncrystallizing or supercooled systems. For example, Figure 3 represents  $\Delta S$  and  $S_c$  for a hypothetical supercooled PE liquid, which is difficult to obtain experimentally owing to its very rapid crystallization rate. Over the entire temperature range,  $S_c$  accounts for only a portion ( $\sim 50\%$ ) of  $\Delta S$ , and the remainder must be  $S_v$ . This could be demonstrated in other systems (polystyrene or polypropylene, for example) by comparing the atactic, noncrystallizing polymer with its isotactic, crystallizing form.

Since in the present context  $T_2$  (calorimetric) and  $T_0$  (Vogel) are important parameters for estimating  $\Delta S$  and  $S_c$ , it is appropriate to review the arguments regarding their relationship to each other. Although both of these have now been shown to be fictive temperatures,<sup>1,23</sup> they may still give qualitative information on molecular motions in polymer liquids. We have suggested that vinyl-type polymer chains that have substituents whose internal motions can contribute measurably to  $\Delta C_p$  (hence to  $\Delta S$ ) in going into the liquid state will always show  $T_2 < T_0$  since these substituent motions are not directly related to rotation about main-chain bonds, and it is the latter that determines  $S_c$ . Thus  $T_2 < T_0$  for polystyrene<sup>33</sup> but  $T_2 \approx T_0$  for linear polyethylene,<sup>19</sup> poly(ethylene oxide),<sup>20</sup> po-

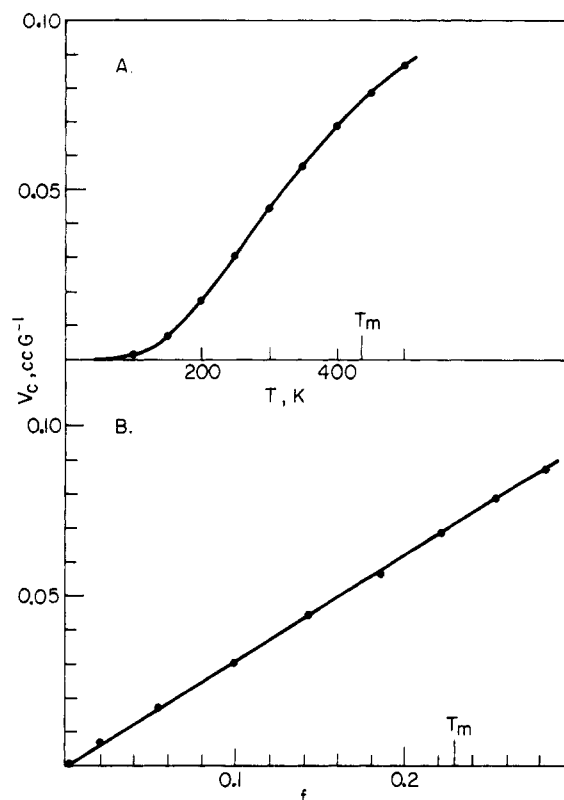


Figure 5. (A) Conformational volume as a function of temperature. (B) Conformational volume as a function of  $f$ , the fraction of flexed bonds by eq 5B.

lypropylene,<sup>33</sup> and poly(propylene oxide).<sup>20</sup> In the latter two polymers, motion (i.e., rotation) of the methyl substituent apparently does not noticeably enhance  $\Delta C_p$  in going to the liquid state.

Nonvinyl polymers with aromatic rings as part of the main chain show especially large differences between  $T_2$  and  $T_0$ . For poly(ethylene terephthalate),<sup>20</sup>  $T_0 - T_2 \approx 100^\circ$ , and for bisphenol-A polycarbonate<sup>20</sup> with a greater aromatic content,  $T_0 - T_2 \approx 140^\circ$ . These results imply that the enhanced motions of these aromatic rings in the liquid state, while they may contribute to  $\Delta S$ , do not contribute significantly to the overall mobility (i.e.,  $S_c$ ) of the polymer liquid which presumably involves rotations about the more flexible bonds in the chain.

These observations are consistent with a quantitative correlation which we have reported<sup>34</sup> for the glass transition:  $K = dV_g/dT - (dV/dT)_p$ , where  $V_g$  is the specific volume at the glass-liquid boundary and  $V$  is the liquid specific volume at atmospheric pressure. The same value,  $-10^4 K = 17 \pm 1$   $\text{cm}^3 \text{g}^{-1} \text{deg}^{-1}$ , was found for a number of vinyl-type polymers while the polycarbonate gave a lower value,  $-10^4 K = 10.3$ . Recent measurements by Zoller<sup>35</sup> on a highly aromatic polysulfone lead to an even lower value,  $-10^4 K = 6.6$   $\text{cm}^3 \text{g}^{-1} \text{deg}^{-1}$ . Apparently the magnitude of  $K$  is related to the weight fraction of rotating segments of the main chain, with a maximum value for vinyl-type polymers where this weight fraction is uniformly unity.

The present work has demonstrated the fallacy in using isolated or unperturbed chain values for the rotational energy  $U$  in statistical mechanical calculations of  $S_c$  or other conformational properties of the real polymer liquid. In the previous paper,<sup>1</sup> a relatively small (5%) change in  $T_0$  was reported for poly(vinyl acetate) liquid between atmospheric pressure and 1000 bars, indicating a correspondingly small perturbation on  $U$  by pressure. However, by extrapolating into the "negative pressure" region and

approaching the conditions of the isolated chain, we have since demonstrated<sup>27</sup> a far greater decrease in  $T_0$  (hence in  $U$ ) for a number of vinyl polymers including PE. It is now apparent that our value of  $U$  is not the "average" between  $U_g$  and  $U_\infty$ , as suggested earlier,<sup>1</sup> but rather it is  $U_g$ , which has been appreciably increased by intermolecular interactions.

The fraction of flexed bonds  $f$  is more sensitive to the value of  $U$  than is  $S_c$  and might be more amenable to experimental test or direct measurement. For example, in PE at  $T_m = 414$  K by eq 5B, our treatment gives  $f = 0.23$ , but with  $U = 500$  cal/mol (the isolated chain value),  $f = 0.45$ , about twice as large. In this connection, it should be noted that measurements of the molecular dimensions of liquid  $n$ -alkanes by small-angle neutron scattering when interpreted by the rotational isomeric state model require that  $U \simeq 500$ – $600$  cal/mol, agreeing with the unperturbed value for linear polyethylene.<sup>24</sup> However, there may still be uncertainties in the relationship between molecular dimensions and  $U$ , or more specifically, the fraction of flexed bonds. Preliminary calculations by Cook and Moon<sup>36</sup> have shown that variations in the rotational angle about the  $t$ ,  $g^+$ ,  $g^-$  minima could alter the end-to-end distance beyond what the rotational isomeric model would predict, tending to shorten an extended chain or lengthen a compacted one. This "flexibility" would make a low value of  $f$  appear to be higher when the end-to-end distance is interpreted by the unmodified rotational isomeric state model. Further calculations are required to determine the magnitude of this effect and whether the differing  $U$  and  $f$  values can thus be reconciled.

In conclusion, it is emphasized that our entire treatment has been based on a three-state rotational model including the "pentane interference". While apparently quite correct for linear polyethylene, this model may be more of an approximation when applied to other polymers (cf. discussions in ref 3 and 4). However, the conformational properties of the bulk polymer liquid may still be dominated by a single value of  $U$ , which in the present context is uniquely defined by the Vogel  $T_0$ . This in turn can be derived from the liquid mobility as manifested, for example, by the Newtonian viscosity or the  $\alpha$  process in dielectric relaxation.

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